Math Virtual Learning

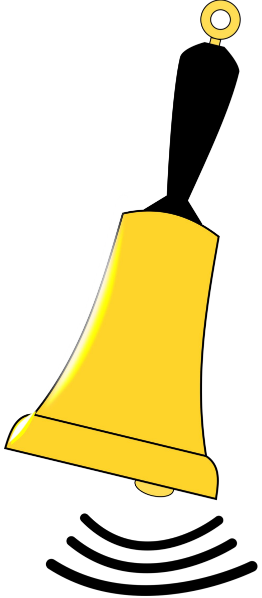
Probability and Statistics

May 1, 2020

Probability and Statistics
Lesson: May 1, 2020

Objective/Learning Target:
Students will be able to differentiate when to use the Central Limit Theorem calculations and when to use Sample calculations

Let's Get Started!



In the US, the height of men is normally distributed. The mean of the average man is 69.1 inches tall with a standard deviation of 2.9 inches.

1. Find the probability that ONE random man is taller than 72 inches.

2. Find the probability that the mean of a sample of 10 men is taller than 72 inches.

3. Find the probability that the mean of a sample of 100 men is taller than 72 inches.

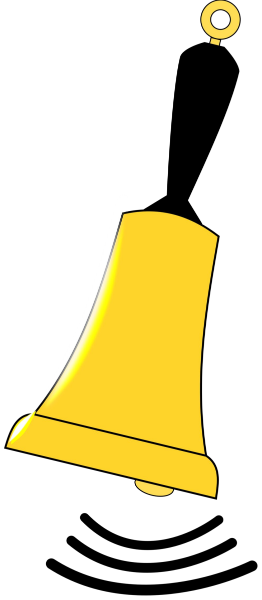
Let’s Get Started!

In the US, the height of men is normally distributed. The average man is 69.1 inches tall with a standard deviation of 2.9 inches.

1. Find the probability that ONE random man is taller than 72 inches.

When it asks about one item, you are dealing with population

$$\frac{72-69.1}{2.9} = .8413 = 84.13\%\ shorter, so\ 100\% - 84.13\% = \mathbf{15.87\%\ taller}$$



Some of you might be thinking, “Why is it 84.13% and not an even 84% (because if the z-score is 1, then according to the Empirical Rule, that should be 50% + 34% = 84%)

Kudos to you for recognizing that!

The reason is that even though the Empirical Rule we learned said 1 Stand. Dev. on each side of the Mean is 68%, the truth is, it is actually 68.27%, but the Empirical Rule rounds it to an even 68% for simplicity. When you take a college Statistics course, your professor will probably make you use the decimal value and the Z-Score Chart that we use also uses the exact decimal values.

Let’s Get Started!

In the US, the height of men is normally distributed. The average man is 69.1 inches tall with a standard deviation of 2.9 inches.

2. Find the probability that the mean of a sample of 10 men is taller than 72 inches.

When it asks about more than one item, you are dealing with a sample

$$\frac{72-69.1}{\frac{2.9}{\sqrt{10}}} = 3.16 = .9992 = 99.92\%\ shorter, so\ 100\% - 99.92\% = \mathbf{.08\%\ taller}$$

There is a VERY small chance (.08%) that if you average samples of 10, that your Mean will end up to be 72 or higher.

Let’s Get Started!

In the US, the height of men is normally distributed. The average man is 69.1 inches tall with a standard deviation of 2.9 inches.

3. Find the probability that the mean of a sample of 100 men is taller than 72 inches.

When it asks about more than one item, you are dealing with a sample

$$\frac{72-69.1}{\frac{2.9}{\sqrt{100}}} = 10 = 1 = 100\%\ \mathit{shorter, so}\ 100\% - 100\% = \mathbf{0\%\ \mathit{taller}}$$

So how can you have a Z-Score of 10? Doesn’t it stop at like 3.5 or something?
Once you reach a Z-score that is off the graph, it means that there is 0 probability that you will get that result.
In our example, there is 0% chance that if we take samples of 100 that we will get a Mean of 72 inches or taller. (given that it is a true RANDOM sample)

Z-score Recap

Things to Remember:

- We used z-scores to find percentages of data that fall into a certain range
- We used the formula

$$Z = \frac{\overset{\text{Score}}{x} - \overset{\text{Mean}}{\mu}}{\underset{\text{SD}}{\sigma}}$$

- We used this Conversion Chart to translate a z-score into a percentage
- This was for **specific data points** of a **specific population** that followed a normal curve

Central Limit Theorem Recap

Things to Remember…

- The Central Limit says that even if we can't calculate data on a population, we can calculate data on the **MEANS** of random samples taken over & over again.
  - Example - If we sampled the weight of 25 boys at a middle school, the data may not be normal. BUT if we sample another 25 and then another 25 and then another 25 over and over again, the MEANS of all those samples would be normal
- We used a slightly different z-score formula

DESIRED VALUE

MEAN

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

STANDARD DEVIATION

SAMPLE SIZE

Practice Differentiating

Decide which formula should be used for each of the statistical measures desired.

Score Mean

$$z = \frac{x - \mu}{\sigma}$$

SD

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

1. A population of watermelon were found to have a mean of 21 seeds with a standard deviation of 3. What is the probability that a random sample will have an average of less than 16 seeds?
2. A random sample of watermelons was tested to find the number of seeds in each watermelon. The sample was found to be normal with a mean of 21 and a standard deviation of 3. What percent of the watermelon had between 16-19 seeds?
3. A population of watermelon were found to have a mean of 21 seeds with a standard deviation of 3. What is the probability that a random sample will have an average of 16-19 seeds?

Practice Differentiating ANSWERS

1. A population of watermelon were found to have a mean of 21 seeds with a standard deviation of 3. What is the probability that a random sample of 30 watermelon, will have an average of less than 16 seeds?

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Because we are focused on the mean of a sample of a population

2. We sampled a population of watermelon to find the number of seeds in each watermelon. It was found that the distribution was normal with a mean of 21 and a standard deviation of 3. What percent of the watermelon have between 16-19 seeds?

Score Mean

$$z = \frac{x - \mu}{\sigma}$$

SD

Because we are focused on individual values of a population

3. A population of watermelon was found to have a mean of 21 seeds with a standard deviation of 3. What is the probability that a random sample of 30 will have an average of 16-19 seeds?

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Because we are focused on the mean of a sample of a population

Practice Problem #1: Pre-thinking

A local research company investigated the number of times a person makes a trip to a grocery store in a given week. The population was found to have a mean of 8 with a standard deviation of 5. What is the probability that a sample of 100 randomly selected people will visit the store a mean of 7-9 times per week?

A. Is the standard deviation of 5 representing the population or a sample?
B. Is the statistical measure needed asking about individual data points or a mean of a sample?
C. Which z-score formula should you use?

Score

Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Practice Problem #1: Pre-thinking ANSWERS

A local research company investigated the number of times a person makes a trip to a grocery store in a given week. The population was found to have a mean of 8 with a standard deviation of 5. What is the probability that a sample of 100 randomly selected people will visit the store a mean of 7-9 times per week?

A. Is the standard deviation of 5 representing the population or a sample? POPULATION
B. Is the statistical measure needed asking about individual data points or a mean of a sample? MEAN OF A SAMPLE
C. Which z-score formula should you use?

Score
Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}}$$

Practice Problem #1: Solve the Problem

A local research company investigated the number of times a person makes a trip to a grocery store in a given week. The population was found to have a mean of 8 with a standard deviation of 5. What is the probability that a sample of 100 randomly selected people will visit the store a mean of 7-9 times per week?

CLICK HERE FOR THE Z-SCORE TO PERCENTILE CONVERSION CHART

Practice Problem #1: Solve the Problem ANSWER

A local research company investigated the number of times a person makes a trip to a grocery store in a given week. The population was found to have a mean of 8 with a standard deviation of 5. What is the probability that a sample of 100 randomly selected people will visit the store a mean of 7-9 times per week?

$$z = \frac{7-8}{\frac{5}{\sqrt{100}}} = \frac{-1}{\frac{5}{10}} = \frac{-1}{.5} = -2$$

-2 = 0.0228 = 2.28%

98.21 - 2.28 = 95.93%

$$z = \frac{9-8}{\frac{5}{\sqrt{100}}} = \frac{1}{\frac{5}{10}} = \frac{1}{.5} = 2$$

2 = 0.9821 = 98.21%

There is a 95.93% chance that the sample will have a mean between 7-9

Practice Problem #2: Pre-thinking

Esmerelda monitored the growth of 32 sage plants. After one week, the mean growth of the plants was 4 cm with a standard deviation of 0.6. Assuming the sample data was normal, what percentage of the plants grew 3 cm or less?

A. Is the standard deviation of 0.6 representing the population or a sample?
B. Is the statistical measure needed asking about individual data points or a mean of a sample?
C. Which z-score formula should you use?

$$Z = \frac{\overset{\text{Score}}{x} - \overset{\text{Mean}}{\mu}}{\underset{\text{SD}}{\sigma}}$$

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Practice Problem #2: Pre-thinking ANSWERS

Esmerelda monitored the growth of 32 sage plants. After one week, the mean growth of the plants was 4 cm with a standard deviation of 0.6. Assuming the sample data was normal, what percentage of the plants grew 3 cm or less?

A. Is the standard deviation of 0.6 representing the population or a sample? SAMPLE

B. Is the statistical measure needed asking about individual data points or a mean of a sample? INDIVIDUAL DATA POINTS

C. Which z-score formula should you use?

Score

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Practice Problem #1: Solve the Problem

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Practice Problem #2: Solve the Problem ANSWER

Esmerelda monitored the growth of 32 sage plants. After one week, the mean growth of the plants was 4 cm with a standard deviation of 0.6. Assuming the sample data was normal, what percentage of the plants grew 3 cm or less?

$$z = \frac{3-4}{0.6} = \frac{-1}{0.6} = -1.67$$

-1.67 = 0.2514 = 25.14%

25.14% of the plants grew 3 cm or less

Practice Problem #3: Pre-thinking

Adisa read an article about a study to see how much time teens between the ages of 15-19 spent talking with their guardians. It was found that the population spent an mean of 13 hr/week in conversation with their guardians? The population had a standard deviation of 6. If 30 kids aged 15-19 were sampled at random, what is the probability that they spent more than a mean of 16 hrs in conversation with their guardians?

A. Is the standard deviation of 6 representing the population or a sample?
B. Is the statistical measure needed asking about individual data points or a mean of a sample?
C. Which z-score formula should you use?

$$Z = \frac{\overset{\text{Score}}{x} - \overset{\text{Mean}}{\mu}}{\underset{\text{SD}}{\sigma}}$$

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A. Is the standard deviation of 6 representing the population or a sample?
   POPULATION
B. Is the statistical measure needed asking about individual data points or a mean of a sample? MEAN OF A SAMPLE
C. Which z-score formula should you use?

Score

Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD

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$$z = \frac{16-13}{\frac{6}{\sqrt{30}}} = \frac{3}{\frac{6}{5.48}} = \frac{3}{1.09} = 2.75$$

2.75 = 0.9970 = 99.7%

This is the amount who spent 16 hours or less. We want MORE than 16 hours

100 - 99.7 = 0.3%

There is a 0.3% chance that the sample would have an average of more than 16 hours